In a nutshell: The bisection method

Given a continuous real-valued function f of a real variable defined on the interval $[a_0, b_0]$ where $f(a_0)$ and $f(b_0)$ have opposite signs and neither is zero, the intermediate-value theorem guarantees that there is a root on that interval. This algorithm uses iteration, bracketing and averaging to approximate the root. The intermediate-value theorem is used to guarantee the existence of the root.

Parameters:

$\varepsilon_{\text{step}}$	The maximum error in the value of the root cannot exceed this value.
ε_{abs}	The value of the function at the approximation of the root cannot exceed this value.
N	The maximum number of iterations.

1. Let $k \leftarrow 0$.
2. Given that we have constrained the root to $[a_k, b_k]$, if $b_k - a_k < \varepsilon_{\text{step}}$ and $\min\{|f(a_k)|, |f(b_k)|\} < \varepsilon_{\text{abs}}$, we are done, and return whichever end-point has the smallest absolute value, returning either in the very unlikely case that $|f(a_k)| = |f(b_k)|$.
3. If $k > N$, we have iterated N times, so stop and return signalling a failure to converge.
4. Let $m_k \leftarrow \frac{a_k + b_k}{2}$.
    a. If $f(m_k) = 0$, we are done, and return m_k.
    b. If $f(m_k)$ and $f(a_k)$ have the same sign, let $a_{k+1} \leftarrow m_k$ and $b_{k+1} \leftarrow b_k$;
    c. otherwise, $f(m_k)$ and $f(b_k)$ must have the same sign, so let $a_{k+1} \leftarrow a_k$ and $b_{k+1} \leftarrow m_k$.
5. Increment k and return to Step 2.

Convergence

The maximum error at any step is $b_k - a_k$, so with each step, the maximum error is halved. Thus, if h is the error, the error decreases according to $O(h)$.